

## Assignment 3 for Formal Modeling (Winter Semester 2025)

### Symbolic Summation and the modeling of sequences

Carsten.Schneider@risc.jku.at

Deadline: January 28, 2025

For this assignment carry out one of the following two tasks.

#### Task 1 Implement polynomial summation.

Let  $\mathbb{Q}[x]$  be the polynomial ring in the variable  $x$  with coefficients from the rational function field  $\mathbb{Q}$ . Implement two functions `PolynomialTelescopingi(f)` where both produce for the input  $f \in \mathbb{Q}[x]$  the output  $g \in \mathbb{Q}[x]$  with

$$g(x+1) - g(x) = f(x). \quad (1)$$

1. For the function  $i = 1$  use the basis transformation between the standard basis and the falling factorial basis.
2. For the function  $i = 2$  use the degree bounding lemma and solve the underlying system of linear equations.

Finally, implement the corresponding functions `PolynomialSummationi(f, a, b)` which computes

$$\sum_{x=a}^b f(x)$$

by using the solution  $g(x)$  of (1) computed by `PolynomialTelescopingi(f)`.

Your implementation can be carried out with your preferred computer algebra system (like Mathematica, Maple or Sage). The following files should be submitted:

1. A file with your documented code
2. together with file that contains concrete examples that can be carried out in your chosen CA system (e.g., a Mathematica notenook if you choose Mathematica); in particular, the efficiency should be compared between both methods.
3. A pdf-file where the computations (examples) from step 2 are documented explicitly.

#### Task 2: Implement the basic functionality of the term algebra `SumProd( $\mathbb{Q}(x)$ )`.

Let  $\mathbb{G} := \mathbb{Q}(x)$  be the rational function field in the variable  $x$  defined over the rational numbers  $\mathbb{Q}$ .

1. Implement the term algebra  $\text{SumProd}(\mathbb{G})$  together with the evaluation function  $\text{ev} : \text{SumProd} \times \mathbb{N} \rightarrow \mathbb{Q}$  from Section 2. More precisely, given  $A \in \text{SumProd}(\mathbb{G})$  with expressions as defined in Section 2 and given  $n \in \mathbb{N}$ , your function  $\text{ev}(A, n)$  should evaluate the expression accordingly.
2. Furthermore implement the function

$$\text{ReducedRepresentation} : \text{SumProd}(\mathbb{G}) \rightarrow \text{SumProd}(\mathbb{G})$$

which transforms an expression from  $\text{SumProd}(\mathbb{G})$  to the reduced representation; see Definition 2.8. Note: a transformation to the sum-product reduced representation is not necessary.

Your implementation can be carried out with your preferred computer algebra system (like Mathematica, Maple or Sage). Hint: for the implementation of **ReducedRepresentation** you are allowed to use the standard field/ring operations (together with functions working on variables) of your available computer algebra system.

The following files should be submitted:

- A file with your documented code
- together with file that contains concrete examples that can be carried out in your chose CA system (e.g., a Mathematica notenook if you chose Mathematica); in particular, the correctness of the evaluations should be demonstrated.
- A pdf-file where the computations (examples) from step 2 are documented explicitly.