FORMAL MODELING

How to Write a Mathematical Article

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Some general remarks

- Every author has her/his own style and preferences how to write an article.
- The structure strongly depends on the topic/message of the article.

⇒ **There is no final word how one should write a mathematical article...**

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The outline

1. The general form using latex (with the LNCS-style)

Some (hopefully) useful remarks (based on my experience):

2. How to write the mathematical content (just math!)

3. Presenting the main content (with explanations)

4. General/personal hints/tips

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1. The general form using latex (with the LNCS-style)

```
\documentclass[english, runninghead, a4paper]{llncs}
... %define macros/load extra packages...
\begin{document}

\title{Find a catchy title\thanks{Supported by the Austrian FWF....}}

\author{Carsten Schneider}
\institute{Research Institute for Symbolic Computation (RISC)\\ Johannes Kepler University Linz\\
    Altenberger Str.\ 69, 4040 Linz, Austria\\ \email{cschneid@risc.jku.at}}
\maketitle

\begin{abstract}
    The abstract should briefly summarize the contents; 150–250 words.
    \keywords{First keyword \and Second keyword \and Another keyword.}
\end{abstract}

\section{Introduction}\label{Sec:Introduction}

\section{Further sections...}\label{Sec:NextSection}

\section{Conclusion}\label{Sec:Conclusion}

\begin{credits}
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1. The general form using latex (with the LNCS-style)

```
\documentclass[english, runninghead, a4paper]{llncs}
... %What ever you need...
\begin{document}
```

The document style

- Each journal/conference proceeding has its own style whose rules one should/must follow.
- Warning: proceedings often have rigorous page limits!
- However, the structure (up to some technical details) is always the same.
- In the *PS Formal Modelling* we require the Springer style LNCS.
  https://www.springer.com/gp/computer-science/lncs/
  conference-proceedings-guidelines

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1. The general form using latex (with the LNCS-style)

```
\title{Find a catchy titile\thanks{Supported by the Austrian FWF....}}
```

The title

- **The first hurdle:** 90% of the potential readers will check only the title.
- A good title should
    - □ be simple and direct;
    - □ be attractive and informative;
    - □ contain the most important keywords;
      (domain/objects, problem/theorem/conjecture, solution/method);
    - □ be not too long or short (or cryptic);
    - □ avoid nonstandard abbreviations and unnecessary acronyms.
- Often the funding agency (e.g. the Austrian FWF) is mentioned via a footnote.

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The author(s) and the institution(s) with email(s)

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The abstract (and keywords)

- **The second hurdle**: Among the 10% who passed the title, 90% will quit after reading the abstract.
- Just plain text: avoid formulas, citations, and references to the text below.
- Simple sentences, avoid technical words (for the non-expert).
- Why should the reader continue reading: explain what you advertised with the title. What is special?
- Summarize the main content/message of the article.

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1. The general form using latex (with the LNCS-style)

```latex
\section{Introduction}
```

The introduction (1–2 pages $+\epsilon$)

- **The final hurdle**: Among the remaining readers only 10% will proceed...
- It should present the state-of-the-art citing relevant/related work.
- It should connect the content of this article with the known results
- It should specify the main result:
    - □ ideally, a precise statement/specification/problem description;
    - □ if too complicated, a simplified version in prose;
    - □ if possible, a concrete (short!) example to illustrate the result.
- The last paragraph should sketch the outline of the remaining sections:

```latex
The outline of the article is as follows. In Section~\ref{Sec:NextSection} the basic
definitions and the detailed problem description is given.... The conclusion is given
in Section~\ref{Sec:Conclusion}.
```

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1. The general form using latex (with the LNCS-style)

```
\section{Further sections...}
```

The main content

- The structure depends on the concrete context/the preferences of the author.
- Some hints will be given later...
- Of great help are environments (available in LNCS):

  ```
  We follow the definitions given in~\cite{PS:19}.
  \begin{definition}\label{Def:DR}
    A difference ring $(\set A,\sigma)$ is a ring $\set A$ equipped with a ring automorphism $\sigma:\set A\to\set A$.
  \end{definition}

  \begin{lemma}\label{Lemma:FirstResult}

  ...
  \end{lemma}

  Using Lemma~\ref{FirstResult} we can now prove our first main result.
  \begin{theorem}
  Article writing is fun.
  \end{theorem}
  ```

- and macros (see, e.g., https://arxiv.org/)

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1. The general form using latex (with the LNCS-style)

`\section{Conclusion}`

The conclusion

- It summarizes the main results of the article (extra notions are allowed that occur within the main text).
- However, readers might jump directly from the introduction to the conclusion. Thus avoid too many technical words.
- An outlook is given. E.g., the following questions can be addressed:
  - What can be done next?
  - What are possible (future) applications?
  - Are there conjectures/unanswered problems?
  - How can the presented ideas be generalized/extended?

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\begin{credits}
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\end{credits}
```

The acknowledgment

- This part is optional.
- If appropriate, one can thank
  - □ colleagues (advisors) for fruitful discussions;
  - □ referees for careful reading and valuable comments;
  - □ funding agencies (if the thanks command in the title is excluded by the editor).

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\bibliography{mybibliography}
\end{document}
```

The bibliography

- Store all bib-items in `mybibliography.bib`. E.g.,

  ```
  @incollection {PS:19,
      AUTHOR = {Paule, Peter and Schneider, Carsten},
      TITLE = {Towards a symbolic summation theory for unspecified sequences},
      BOOKTITLE = {Elliptic integrals, elliptic functions and modular forms inquantum field theory},
      SERIES = {Texts Monogr. Symbol. Comput.},
      PAGES = {351–390},
      PUBLISHER = {Springer, Cham},
      YEAR = {2019}
  }
  ```

- Bib-items can be copied from many places:
  - https://mathscinet.ams.org/mathscinet/index.html
  - https://dblp.uni-trier.de/
  - https://zbmath.org/authors/

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\section{Appendix}
...
\section{Appendix 2}
...
```

The appendix

- This part is optional.
- Depending on the convention, it can be put before or after the bibliography.
- Move aspects to the appendix that would destroy the main flow of the article, but which are relevant for the interested reader. Typical examples are:
  - □ A detailed proof of a statement that is needed in the article (but which is irrelevant to understand the main result).
  - □ A detailed example/demo.
  - □ Further aspects that are related but that do not fit directly to the main topic.
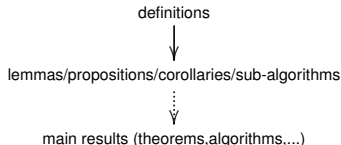
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2. **How to write the mathematical content (just math!)**

3. Presenting the main content (with explanations)

4. General/personal hints/tips

2. How to write the mathematical content (just math!)

[Note: here you see my personal opinions/experiences]

1. First produce correct mathematics in bottom-up style:

<div align="center">

definitions

↓

lemmas/propositions/corollaries/sub-algorithms

⋮

main results (theorems,algorithms,...)

</div>

2. Afterwards check (and, if necessary, go back to 1):
   - □ correctness;
   - □ completeness:
       - Are there details missing in the proof?
           - → Maybe an extra lemma is in place?
       - Can one easily generalize some results (without extra pain)?
       - Are all notions properly defined?
           - → Insert extra definitions...
   - □ Can one simplify the proofs?
       - Often the definitions are clumsy.
       - Being slightly less general might simplify the proofs dramatically (but sometimes the opposite is true...).

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3. Presenting the main content (with explanations)

There are two main approaches.

1. proof oriented: bottom-up construction

   definitions → lemma → ... → lemma/proposition → main theorems/algorithms

2. reader friendly: top down construction

   problem specification → theorem → lemma/proposition → ... → basic constructions/definitions

   The reader should be motivated: why do we need all these technical proofs/constructions to obtain the main result?

Problem: (1) is rather technical, but (2) is often not feasible.

3. Presenting the main content (with explanations)

Often the following combined/hybrid version (or variants of it) is appropriate:

1. Introduction: specify the problem/state the main results. If this is too involved, present simplified versions in text form; use examples for illustration.

3. Presenting the main content (with explanations)

Often the following combined/hybrid version (or variants of it) is appropriate:

1. Introduction: specify the problem/state the main results. If this is too involved, present simplified versions in text form; use examples for illustration.
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3. Finally, apply the bottom-up approach in the remaining section(s).
   But in the beginning of each section describe in simple words, what one will accomplish and how this is related to the original problem.

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Rather time consuming, but it pays off (concerning readability):

Construct one (or more) running example(s) that illustrate the full construction

- it should be compact but not trivial (the highlights/results of the article must be illustrated);
- it should start with the basic definitions and end, e.g., with the output of your algorithm.

The outline

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4. **General/personal hints/tips**

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- All notions/shortcuts/functions must be defined before they can be used.

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- **The devil is in the detail, or: typos prove the existence of the devil!**

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  - □ Check the text with different focuses:

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- All notions/shortcuts/functions must be defined before they can be used.
- **The devil is in the detail, or: typos prove the existence of the devil!**
  - □ Read the text many times (loud).
  - □ Put the text aside and read it some days later.
  - □ Check the text with different focuses:
    - just parse formulas for syntactical correctness (brackets etc);
    - read the text but ignore formulas;
    - compare related formulas and check inconsistencies;
    - check if all objects are defined when they arise the first time;
    - check if all existing results are properly cited;
    - check the citation list (spelling);
    - check if you cited all articles that are relevant;
    - ...

Some general remarks

- Every author has her/his own style and preferences how to write an article.
- The structure strongly depends on the topic/message of the article.

⇒ **There is no final word how one should write a mathematical article...**

Good luck for an excellent article!