6^{th} practice sheet Experimental Design

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17. Consider the following design in five treatments, ten row-blocks and four column-blocks, and resulting data:

row blocks	column blocks 1	2	3	4
1	trt 1, $y = 32$	trt 2, $y = 29$	trt 3, $y = 31$	trt 4, $y = 24$
2	trt 2, $y = 30$	trt 3, $y = 31$	trt 4, $y = 25$	trt 5, $y = 34$
3	trt 3, $y = 23$	trt 4, $y = 21$	trt 5, $y = 29$	trt 1, $y = 24$
4	trt 4, $y = 26$	trt 5, $y = 32$	trt 1, $y = 32$	trt 2, $y = 30$
5	trt 5, $y = 35$	trt 1, $y = 31$	trt 2, $y = 33$	trt 3, $y = 30$
6	trt 1, $y = 34$	trt 2, $y = 28$	trt 3, $y = 32$	trt 4, $y = 24$
7	trt 2, $y = 22$	trt 3, $y = 22$	trt 4, $y = 17$	trt 5, $y = 23$
8	trt 3, $y = 22$	trt 4, $y = 23$	trt 5, $y = 28$	trt 1, $y = 24$
9	trt 4, $y = 33$	trt 5, $y = 41$	trt 1, $y = 39$	trt 2, $y = 36$
10	trt 5, $y = 37$	trt 1, $y = 32$	trt 2, $y = 34$	trt 3, $y = 29$

Assume that row-blocks, column-blocks, and treatments can be regarded as additive effects (i.e., that no two of them interact).

- Using a computer, calculate the least-squares estimates of all 10 treatment parameter differences $\tau_i - \tau_j$ under a model in which row-blocks and column-blocks are each assumed to have fixed effects. Compute the common margin of error for these differences corresponding to 95% confidence (i.e., half the length of the usual 95% confidence interval).
- Calculate the inter-block estimates of the 10 treatment parameter differences under a model in which row-blocks are assumed to be random but column-blocks are assumed to be fixed. Compute the common margin of error for these differences corresponding to 95% confidence (again, based only on inter-block information).

18. Consider an extended 8-block version of the radon detector experiment of exercise 16. The design and resulting data are given in the following table:

chamber	detector type			
session	A	B	C	D
1	6.11	-	5.95	5.82
2	6.70	6.22	-	5.97
3	6.60	6.11	6.52	-
4	-	6.22	6.54	6.18
5	6.34	-	6.20	6.06
6	6.77	6.30	-	6.02
7	6.55	6.09	6.48	-
8	-	6.04	6.24	5.98

(a) Assuming that chamber sessions are represented by fixed effects, compute the least-squares estimate of $\tau_1 - \tau_2$, and compute MSE for the full model (blocks and treatments). Use statistical software to work this and all parts of this exercise.

(b) Assuming that chamber sessions are represented by random effects, construct a separate inter-block estimate of $\tau_1 - \tau_2$ using only the chamber session totals, and calculate the MSE for this model.

(c) Use the estimates and MSEs of the two models you have fit to construct a single combined estimate of $\tau_1 - \tau_2$ appropriate for the random-sessions model.

(d) Suppose your estimates of the unit and block variances are exactly correct, rather than estimates. Pretending this is true, what is the standard deviation of each of the following?

- the intra-block (i.e., fixed-block) estimate of $\tau_1 - \tau_2$
- the inter-block estimate based on block totals
- the combined estimate

19. A cell biologist designs an experiment to study the joint effect of exposure to ionizing radiation and a particular toxic chemical to the survival of marrow stem cells taken from a specific strain of mouse. He designs a factorial experiment to study all combinations of three radiation doses (including no radiation) and four concentrations of the chemical (including zero concentration).

Suppose now that the experiment to be executed as a CRD, 36 cell cultures are prepared, each consisting of a standard number of cells in medium in a petri dish, and the same numbers of petri dishes are to be allocated to all treatments.

Derive the noncentrality parameter associated with the test for a radiation main effect under this design. If $\sigma = 3$ and $\mathsf{E}(\bar{y}_{3..}) - 1 = \mathsf{E}(\bar{y}_{2..}) = \mathsf{E}(\bar{y}_{1..})$, what is the power of this test performed at level 0.05 under this design?

20. An experimenter wants to understand the effect of a three-level factor („factor 1") on a response. She also has some interest in the effects of „factor 2" and „factor 3", each of which has three levels, and can be used jointly with factor 1 to define treatments, but these factors are not the focus of her current research. She is considering three different experimental designs, each requiring $n = 54$ units:

- Design A: A 3^1 design in factor 1, with each treatment applied to $r = 18$ units.
- Design B: A 3^2 design in factors 1 and 2, with each treatment applied to $r = 6$ units.
- Design C: A 3^3 design in factors 1, 2, and 3, with each treatment applied to $r = 2$ units.

Each design can be executed without blocking.

(a) For each design, what is the expected squared length of a confidence interval for α_1 (full-rank model) if $\sigma^2 = 1$?

(b) For each design, what is the power of the test for $H_0 : \boldsymbol{\alpha} = \mathbf{0}$ if, in fact, $\alpha_1 = \alpha_2 = 0.1$ and $\sigma^2 = 1$?