5^{th} practice sheet Experimental Design

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13. Consider the following partially balanced incomplete block design, a PBIBD(2) for comparing eight treatments in eight blocks of size 4 units each:

1	5	2	6	3	7	4	8
2	6	7	3	8	4	1	5
3	7	8	4	1	5	6	2
4	8	1	5	6	2	7	3

   (a) For each treatment, identify the first- and second-associates.

   (b) Determine the values of all design parameters:
   $t, b, k, t_1, t_2, \pi_1, \pi_2, p_{11}, p_{12}, p_{22}, q_{11}, q_{12}, q_{22}$

14. A researcher wants to conduct a blocked experiment to compare five treatments, but operational constraints require that the blocks be of size 3.

   (a) She likes the idea of performing the experiment in five blocks since that would allow her to assign each treatment to three different units in the experiment. Can a BIBD for this experiment be constructed under these conditions? Prove your answer.

   (b) Construct a BIBD for this experiment using ten blocks (not five). Fully specify your design by making a table with 10 rows for blocks, with three entries in each row for the treatments to be included in that block.

   (c) Suppose we think of this experiment with reference to the model:

   $$y_{ij} = \alpha + \beta_i + \tau_j + \varepsilon_{ij}$$

   where y is the response, β's are block effects, and τ's are treatment effects. Suppose that, in fact (although we don't know it as experimenters),

   $$\tau_1 = \tau_2 = -1 \qquad \tau_3 = 0 \qquad \tau_4 = \tau_5 = 1 \qquad \sigma = 2$$

   Given this information, completely characterize the distribution of the F-statistic that would be used to test for equality of treatments. (That is, specify the distribution including numerical values of all parameters.)

   (d) Continuing with part (c), suppose the researcher had been able to execute a randomized complete block design in ten blocks, rather than the BIBD we have been discussing. Note that this would have been a larger experiment, since each block would contain five units. Using the same model information provided in part (c), give a complete characterization of the distribution of the F-statistic that would be used to test for equality of treatments in this case.

15. Consider a BIBD in $b = 20$ blocks of size $k = 2$ units each for comparing $t = 5$ treatments. For this design:

   (a) Apart from a factor of σ^2, what is the sampling variance of $\widehat{\tau_1 - \tau_2}$?

   (b) What t-value would be used to construct a 95% two-sided confidence interval for $\tau_1 - \tau_2$?

   (c) If $\tau_1 = \tau_2 = 0$, $\tau_3 = 1$, $\tau_4 = \tau_5 = 2$ and $\sigma^2 = 2$, what is the noncentrality parameter of the distribution of he test statistic for

$$H_0: \quad \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5$$

16. A consumer products testing laboratory performed a study to compare four varieties of home radon detectors. Trials were performed in a laboratory chamber that was large enough for simultaneous testing of only three units. Units processed together in the chamber can safely be assumed to experience very similar exposures, but there may be some variation in achieved chamber conditions among the chamber operation „sessions“. The design and resulting data (expressed in a unitless efficiency measure) are given in the following table:

chamber	detector type			
session	A	B	C	D
1	6.11	-	5.95	5.82
2	6.70	6.22	-	5.97
3	6.60	6.11	6.52	-
4	-	6.22	6.54	6.18

   (a) Identify the treatments and units in this experiment.

   (b) Use a computer package such as R to compute sums of squares for blocks, treatments after accounting for blocks, residuals, and corrected total. Do this by fitting two models, one containing terms for only chamber session effects, and one containing terms representing both chamber sessions and detector effects, and assemble the required information from the two fits.

   (c) Perform an F-test for equality of detector types.

   (d) Assuming that blocks constitute fixed effects, derive the least-squares estimates and standard errors of the six treatment differences:

$$\tau_1 - \tau_2 \qquad \tau_1 - \tau_3 \qquad \tau_1 - \tau_4 \qquad \tau_2 - \tau_3 \qquad \tau_2 - \tau_4 \qquad \tau_3 - \tau_4$$