4^{th} practice sheet Experimental Design

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9. Consider a Latin square design for comparing three treatments, specifically:

1	2	3
2	3	1
3	1	2

where rows in this figure correspond to one kind of block, columns to another kind of block, and numbers refer to the associated treatment.

   (a) How many degrees of freedom are available for estimating σ^2 in this design, if row-blocks, column-blocks, and treatments are assumed to have additive treatments (i.e., no interactions)?

   (b) What is $\mathbf{H}_1\mathbf{X}_2$ for this design? Are the least-squares estimates of estimable contrasts of τ's the same for this design as they would be for a completely randomized design with three units assigned to each treatment? Why or why not?

   (c) Apart from the factor of σ^2, what is $\mathsf{Var}(\widehat{\tau_1 - \tau_2})$ for this design?

10. Suppose the investigator who was planning to use the design in exercise (9) decided at he last minute that she was not interested in treatment 3 after all. However, she had already arranged to use units which would be appropriate for a Latin square of order 3. So, she considered simply not using those units which would have been assigned treatment 3. That is, she considered the experimental design which might be described as:

1	2	-
2	-	1
-	1	2

where as before, rows in the figure correspond to one kind of block, columns to another kind of block, and numbers refer to the associated treatments, but where the units corresponding to the cells containing „-“ were simply not used.

   (a) How many degrees of freedom are available for estimating σ^2 in this design, if row-blocks, column-blocks, and treatments are assumed to have additive treatments (i.e., no interactions)?

(b) What is $\mathbf{H}_1\mathbf{X}_2$ for this design? Are the least-squares estimates of estimable contrasts of τ's the same for this design as thy would be for a completely randomized design with three units assigned to treatments 1 and 2? Why or why not?

(c) What is the design information matrix for this design? Apart from the factor of σ^2, what is $\mathsf{Var}(\widehat{\tau_1 - \tau_2})$ for this design?

11. As part of a study designed to investigate the effects of whole-body X-irradiation on the nervous system of rats, Matsuu et al. (2005) carried out a small experiment involving four experimental treatments. The units in this study were 20 Wistar-Kyoto rats. Treatments numbered 2 through 4 were defined as exposure to a standard dose of radiation, followed by 4, 8, or 24 hours, respectively, before sacrifice and analysis. Group 1 was a control group; rats in his group were housed and handled as those in groups 2-4, but they received no radiation. Upon sacrifice, the adrenal gland of each rat was removed, and the amount of epinephrine in the gland determined, resulting in one data value per rat. Interest in this experiment lay in comparing the groups - especially the control group with each of groups 2 through group 4 - for possible differences in the responses that could be attributed to the treatments. The following table contains the results reported from the experiment:

Treatment			
1	2	3	4
9.934	8.675	10.509	8.829
9.819	10.720	8.067	10.484
10.693	10.040	9.027	8.632
10.106	9.894	9.680	8.352
9.139	11.912	8.967	9.323

Tabelle 1: Epinephrine levels (Grams, g) in rats treated with whole body X-irradiation

(a) Perform the modified Levene test to check for equality of variance in the four treatment groups.

(b) Assuming equal variances in the four treatment groups, use the appropriate method to construct simultaneous confidence intervals for comparing the control condition to each of the other treatment groups; use $\alpha_E = 0.05$.

12. Kocaoz, Samaranayake and Nanni (2005) performed a laboratory experiment to compare the effects of four coatings on the tensile strength of steel reinforcement bars of the type used in concrete structures. Three of the coatings were formed from a common matrix of Engineering Thermoplastic Polyurethane (ETPU), embedded with glass fibers, carbon fibers or aramid fibers, respectively. The fourth coating consisted of ETPU only (i.e. no added fibers) and served as an experimental control. The $N = 32$ specimens (coated bars) were prepared in eight groups of four, with each bar type represented in each of the eight groups. The groups act as the 'blocks' in a randomized complete block design, thus adjusting for systematic trends in environmental factors or testing conditions across time. The bars within each group were prepared in random order.

    The prepared bars were tested (destructively) for strength in a set-up requiring each bar to be anchored in a pipe filled with grout. The bars from a given block were tested together. Since all four bars in a group were tested within a short period of time (1h) it is assumed that the test conditions within a group were similar. Also, for each group, a single batch of cementitious grout was prepared, thus eliminating any variation due to grout differences among the bars within each group.

    Data reported on bar tensile strength are presented in the following table, by block and coating type.

	Coating			
Block	1	2	3	4
1	136	147	138	149
2	136	143	122	153
3	150	142	131	136
4	155	148	130	129
5	145	149	136	139
6	150	149	147	144
7	147	150	125	140
8	148	149	118	145

Tabelle 2: Tensile strength (Kilograms per square inch, ksi) of steel reinforcement bars

    (a) Perform a Tukey one-degree-of-freedom test to check for interaction between the blocks and treatments.

    (b) Using the appropriate method, construct simultaneous confidence intervals for comparing pairs of treatments; use $\alpha_E = 0.05$.