Exercise Sheet 2

To be submitted by email to manuel@kauers.de until **September 2018.**

You are encouraged to use computer algebra systems for solving the exercises below. You may submit a transcript of your session as (part of) your solution.

Task 1 Find all hyperexponential solutions of the following differential equation:

$$2(x-1)x(2x^2+x-2)f''(x)+(4x^4-3x^2-8x+8)f'(x)-(2x^3+x^2-x-4)f(x)=0$$

Task 2 It was shown in class that $V(\operatorname{lclm}(L_1,L_2)) \supseteq V(L_1)+V(L_2)$. Construct a counterexample for the opposite inclusion.

Hint: Choose $\mathcal{F}$ so that it contains a certain linear combination of two solutions of L_1, L_2 but not the solutions themselves.

Task 3 Let $\mathbb{O} = K[\partial_x, \partial_y]$, and let $\mathcal{F}$ be an $\mathbb{O}$-module. Let $f \in \mathcal{F}$ be D-finite and $P \in \mathbb{O}$. Show that $\dim_K \mathbb{O}/\operatorname{ann}(P \cdot f) \leq \dim_K \mathbb{O}/\operatorname{ann}(f)$.

Hint: Consider what happens when you try to compute a Gröbner basis of $\operatorname{ann}(P \cdot f)$ from a Gröbner basis of $\operatorname{ann}(f)$ using FGLM.

Task 4 a) Let $f = \frac{x+y}{x^2+xy+y^3} \in C(x,y)$. Construct $P \in C(x)[D_x] \setminus \{0\}$ and $Q \in C(x,y)[D_x, D_y]$ such that $(P - D_yQ) \cdot f = 0$ by executing one of the creative telescoping algorithms from the lecture step by step.

b) Using the discrete version of creative telescoping, prove the identity

$$\sum_k (-1)^k \binom{2n}{n+k}^3 = \frac{(3n)!}{n!^3}.$$

In this case, you may use a software package for finding suitable P and Q. The task consists in verifying that they are correct, and turning them into a rigorous proof of the identity.