Experimental Design

Unit 7

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10.1 Introduction to split-plot designs (SPD)

Split-plot designs are factorial plans in which a hierarchy of nested experimental units is used, and the factors are not all applied at the same stratum of units. This occurs as the natural consequence of operational constraints in a number of contexts, e.g.,

- where the levels of one factor are easy to change quickly, but changes in the levels of another are much more difficult or time-consuming,
- where treatments are applied to individuals, and this is most easily done by assigning any given individual to only one level of one factor, but measurements representing all levels of another factor are collected on each individual, or
- where the physical material constituting an experimental unit is sequentially divided or subsampled as time progresses, and the levels of different factors are most easily applied to the experimental material at different stages of this process.

Split-plot designs (cont.)
Chemical reactor example:

We want to compare products made using all combinations of 2 factors:

- factor 1: temperature – $l_1 = 3$ different levels
- factor 2: stirring rates – $l_2 = 4$ different levels

i.e. the number of treatments is $t = 12$. Suppose we perform $r = 2$ replications of each treatment, i.e. $n = 24$ runs are performed in the whole experiment.

Only 4 runs may be accomplished in one day and and possible day-to-day variations in the experimental environment suggest that the runs made during a given day should be regarded as a block, in this case we would have $b = 6$ blocks of size $k = 4$.

A BIBD doesn't exist for this setting.

SPD Example (cont.)

Suppose the levels of temperature of the used material cannot be changed quickly, but the levels of the second factor are easy to change.

The only way that k=4 runs can be executed in one day is if all 4 treatments are run at the same temperature, and only changes among the levels of stirring rate are made within a block.

Treat block effects as fixed: 🙁 in our case temperature is completely confounded with blocks (days), there can be no information available from the data on the main effect of temperature.

Treat block effects as random: 😃 if day-to-day differences can be regarded as random, an inter-block analysis can be used to recover information about the fixed main effects associated with temperature.

In this setting, blocks are sometimes called plots, and the experimental design is often called a split-plot design (SPD). blocks = plots.

Split-plot vs. repeated measures experiments

Repeated measures experiments make use of test *subjects* that can only be treated at a single level of one factor, but at all levels of another factor in „repeated" tests. blocks = subjects.

While split-plot studies and repeated measures studies are generally not physically similar, they share a common statistical structure in that one or more treatment factors are confounded with blocks (plots or subjects). We use the „split-plot" label for both types of experiments.

Split-plot experiments differ from other factorial experiments in that there are two (or even more) definitions of the experimental unit.

The entities we are calling blocks are essentially the units to which levels of the among-blocks factor(s) are applied. blocks = „whole plot units".

10.2 SPD(R,B)

It is helpful to think of a split-plot experiment as being carried out in multiple *strata*, or even as a combination of distinct, *nested experiments*.

In our chemical reactor example, the „whole-plot“ experiment is a CRD in which $3r$ days (at this stratum, experimental units) are randomly assigned to three different treatments (levels of the among-block factor).

The „split-plot“ experiment is a complete block design (CBD) in which the 4 runs (now the units) made in each day (now the block) are randomly assigned to the 4 levels of the within-block factor.

We refer to the design of a split-plot experiment organized in this way as a SPD(R,B), indicating that the portion of the experiment executed at the „top“ stratum is a CRD, while that executed at the „bottom“ stratum is a CBD.

10.2.1 A model

We have to use a factorial model and add a term to account for random block-to-block differences. We start with a two-factor experiment in overparameterized form:

$$y_{ijt} = \dot{\alpha}_i + \dot{\beta}_j + (\dot{\alpha\beta})_{ij} + \varepsilon_{ijt} \qquad i = 1, \ldots, l_1; \quad j = 1, \ldots, l_2; \quad t = 1, \ldots, r$$

To the units within each block only one level of the factor represented by $\dot{\alpha}$, but all levels of the factor represented by $\dot{\beta}$, are applied:

$$y_{itj} = \zeta_{i\,t(i)} + \dot{\alpha}_i + \dot{\beta}_j + (\dot{\alpha\beta})_{ij} + \varepsilon_{i\,t(i)\,j} \qquad i = 1, \ldots, l_1; \quad j = 1, \ldots, l_2; \quad t = 1, \ldots, r$$

$i = 2$, $t(2) = 1$	$i = 1$, $t(1) = 1$	$i = 2$, $t(2) = 2$	$i = 3$, $t(3) = 1$	$i = 1$, $t(1) = 2$	$i = 3$, $t(3) = 2$
$j = 3$	$j = 3$	$j = 4$	$j = 1$	$j = 4$	$j = 2$
$j = 1$	$j = 4$	$j = 2$	$j = 3$	$j = 1$	$j = 4$
$j = 4$	$j = 1$	$j = 3$	$j = 2$	$j = 2$	$j = 3$
$j = 2$	$j = 2$	$j = 1$	$j = 4$	$j = 3$	$j = 1$

A model (cont.)

y_{itj} is the response from applying the j-th level of factor 2, taken from the t-th block (plot) in which factor 1 is applied at level i.

The two random elements in this model are:

- $\zeta_{i\,t(i)}$, representing the (inter-)block variation with $\mathsf{E}(\zeta_{i\,t(i)}) = \mu_\zeta$ and $\mathsf{Var}(\zeta_{i\,t(i)}) = \sigma_\zeta^2$
- $\varepsilon_{i\,t(i)\,j}$, representing the unit-within-block variation (intra) with $\mathsf{E}(\varepsilon_{i\,t(i)\,j}) = 0$ and $\mathsf{Var}(\varepsilon_{i\,t(i)\,j}) = \sigma^2$

We now switch to full-rank parameterization described in Section 9.3.

$\mathbf{y}$ is now ordered lexicographically from top to bottom by (i, t, j), i.e. with all l_2 values associated with a block together, and all subvectors representing blocks associated with the same level of the first factor together.

A model (cont.)

We follow the notation introduced in Chapter 9:

$$\mathbf{y} = \mathbf{X}_1\boldsymbol{\zeta} + \mathbf{X}_\alpha\boldsymbol{\alpha} + \mathbf{X}_\beta\boldsymbol{\beta} + \mathbf{X}_{\alpha\beta}(\boldsymbol{\alpha\beta}) + \boldsymbol{\varepsilon}$$

$$\begin{aligned} \mathsf{E}(\boldsymbol{\zeta}) &= \mu_\zeta\,\mathbf{1}_{rl_1} & \qquad \mathsf{Var}(\boldsymbol{\zeta}) &= \sigma_\zeta^2\,\mathbf{I}_{rl_1} \\ \mathsf{E}(\boldsymbol{\varepsilon}) &= \mathbf{0} & \qquad \mathsf{Var}(\boldsymbol{\varepsilon}) &= \sigma^2\mathbf{I} \end{aligned}$$

with $(l_1 - 1)$-vector $\boldsymbol{\alpha}$, $(l_2 - 1)$-vector $\boldsymbol{\beta}$, $(l_1 - 1)(l_1 - 1)$-vector $(\boldsymbol{\alpha\beta})$ and the $(l_1 r)$-vector of random block effects $\boldsymbol{\zeta}$.

The sub-matrices of the design matrix $\mathbf{X}$ are the Kronecker products of simpler matrices

$$\begin{aligned} \mathbf{X}_\alpha &= \mathbf{1}_{l_2} \otimes \mathbf{1}_r \otimes \mathbf{F}^\alpha & \qquad \mathbf{X}_\beta &= \mathbf{F}^\beta \otimes \mathbf{1}_r \otimes \mathbf{1}_{l_1} \\ \mathbf{X}_{(\alpha\beta)} &= \mathbf{F}^\beta \otimes \mathbf{1}_r \otimes \mathbf{F}^\alpha & \qquad \mathbf{X}_1 &= \mathbf{1}_{l_2} \otimes \mathbf{I}_r \otimes \mathbf{I}_{l_1} \end{aligned}$$

10.2.2 Analysis

The intra-block model is based on $\mathbf{y}_1$, the projection of the data onto the orthogonal complement of the space spanned by the columns of $\mathbf{X}_1$:
$\mathbf{y}_1 = (\mathbf{I} - \mathbf{H}_1)\mathbf{y}$ with

$$\mathbf{H}_1 = \frac{1}{l_2}(\mathbf{J}_{l_2} \otimes \mathbf{I}_r \otimes \mathbf{I}_{l_1})$$

with some algebra $\mathbf{y}_1$ simplifies to

$$\mathbf{y}_1 = \mathbf{X}_\beta \boldsymbol{\beta} + \mathbf{X}_{\alpha\beta}(\boldsymbol{\alpha\beta}) + (\mathbf{I} - \mathbf{H}_1)\boldsymbol{\varepsilon}$$

$$\mathsf{E}((\mathbf{I} - \mathbf{H}_1)\boldsymbol{\varepsilon}) = \mathbf{0} \qquad \mathsf{Var}((\mathbf{I} - \mathbf{H}_1)\boldsymbol{\varepsilon}) = \sigma^2(\mathbf{I} - \mathbf{H}_1)$$

With $\mathbf{X}_{\beta,(\alpha\beta)} = \begin{pmatrix} \mathbf{X}_\alpha & \mathbf{X}_{(\alpha\beta)} \end{pmatrix}$ the weighted normal equations are

$$\mathbf{X}^T_{\beta,(\alpha\beta)}(\mathbf{I} - \mathbf{H}_1)\mathbf{X}_{\beta,(\alpha\beta)} \begin{pmatrix} \boldsymbol{\beta} \\ (\boldsymbol{\alpha\beta}) \end{pmatrix} = \mathbf{X}^T_{\beta,(\alpha\beta)}(\mathbf{I} - \mathbf{H}_1)\mathbf{y}_1 = \mathbf{X}^T_{\beta,(\alpha\beta)}(\mathbf{I} - \mathbf{H}_1)\mathbf{y}$$

These are exactly the same as the reduced normal equations for the fixed-block model.

Intra-block analysis

The information matrix associated with this intra-block analysis is

$$\mathcal{I}_{intra} = \mathbf{X}^T_{\beta,(\alpha\beta)}(\mathbf{I} - \mathbf{H}_1)\mathbf{X}_{\beta,(\alpha\beta)} = n\mathbf{I}$$

The variance of the estimate of any linear combination $\mathbf{c}^T \begin{pmatrix} \boldsymbol{\beta} \\ (\boldsymbol{\alpha\beta}) \end{pmatrix}$ is then

$$\mathsf{Var}(\mathbf{c}^T \begin{pmatrix} \hat{\boldsymbol{\beta}} \\ \widehat{(\boldsymbol{\alpha\beta})} \end{pmatrix}) = \mathbf{c}^T \mathcal{I}^{-}_{intra} \mathbf{c}\, \sigma^2 = \frac{\mathbf{c}^T \mathbf{c}}{n}\, \sigma^2$$

Inter-block analysis

The inter-block analysis is based on the transformed data vector $\mathbf{y}_2 = \mathbf{X}_1\mathbf{y}$. With some algebra $\mathbf{y}_2$ reduces to

$$\mathbf{y}_2 = l_2 \left(\mathbf{1}_r \otimes \mathbf{F}^\alpha\right) \boldsymbol{\alpha} + \mathbf{X}_1^T \left(\mathbf{X}_1 \boldsymbol{\zeta} + \boldsymbol{\varepsilon}\right)$$

With $\boldsymbol{\varepsilon}^* = \mathbf{X}_1^T \left(\mathbf{X}_1(\boldsymbol{\zeta} - \mu_\zeta \mathbf{1}_{rl_1}) + \boldsymbol{\varepsilon}\right)$ we may write an inter-block model as:

$$\mathbf{y}_2 = l_2 \mu_\zeta \mathbf{1}_{rl_1} + l_2 \left(\mathbf{1}_r \otimes \mathbf{F}^\alpha\right) \boldsymbol{\alpha} + \boldsymbol{\varepsilon}^*$$

$$\mathsf{E}(\boldsymbol{\varepsilon}^*) = \mathbf{0} \qquad\qquad \mathsf{Var}(\boldsymbol{\varepsilon}^*) = (l_2^2\,\sigma_\zeta^2 + l_2\sigma^2)\mathbf{I}$$

This model provides the basis for comparative inferences that can be made about the main effect for the among-block factor.

The design information matrix associated with this inter-block analysis is

$$\mathcal{I}_{inter} = l_2 \left(\mathbf{1}_r \otimes \mathbf{F}^\alpha\right)^T \left(\mathbf{1}_r \otimes \mathbf{F}^\alpha\right) = n\mathbf{I}$$

Inter-block analysis (cont.)

The variance of the estimate of any linear combination of the elements of $\boldsymbol{\alpha}$ is

$$\mathsf{Var}(\mathbf{c}^T\hat{\boldsymbol{\alpha}}) = \mathbf{c}^T\mathcal{I}_{inter}\mathbf{c}\left(l_2\,\sigma_\zeta^2 + \sigma^2\right) = \frac{\mathbf{c}^T\mathbf{c}}{n}\left(l_2\,\sigma_\zeta^2 + \sigma^2\right)$$

The dual nature of split-plot experiments means that experimental „noise“ is different for some comparisons than for others.

Block-within-factor variation is the noise connected with the among-blocks factor associated with $\boldsymbol{\alpha}$.

Units-within-blocks variation is the noise connected with the within-block factor main effects $\boldsymbol{\beta}$ and the interactions involving within-block factors $\boldsymbol{\alpha\beta}$.

ANOVA decomposition

The split-plot ANOVA decomposition is often written in a unified form. The degrees of freedom and sums of squares for all factorial effects are computed as they are in a CRD and a CBD, but with two residual or error sums of squares computed corresponding to the noise associated with the two strata of the experiment.

We have a whole-plot portion of the experiment corresponding with the inter-block analysis and a split-plot portion corresponding with the intra-block analysis.

The sums of squares of the split-plot stratum of the experiment are as they would be in a CBD with $l_1 r$ blocks and $(l_2 - 1) + (l_2 - 1)(l_1 - 1)$ treatment degrees of freedom corresponding to the treatment parameters $\boldsymbol{\beta}$ and $\boldsymbol{\alpha\beta}$.

In the split-plot stratum the (corrected) total sum of squares is decomposed according to

$$\mathsf{TSS}_{split} = \mathsf{SST}_{\beta} + \mathsf{SST}_{\alpha\beta} + \mathsf{SSB} + \mathsf{SSE}_{split}$$

ANOVA decomposition (cont.)

$$\mathsf{TSS}_{split} = \sum_{itj} (y_{itj} - \bar{y}_{...})^2 \qquad \text{with } \mathsf{df}_{split} = n - 1 = l_1 l_2 r - 1$$

$$\mathsf{SST}_{\beta} = \sum_{j=1}^{l_2} l_1 r (\bar{y}_{..j} - \bar{y}_{...})^2 \qquad \text{with } \mathsf{df}_{\beta} = l_2 - 1$$

$$\mathsf{SST}_{\alpha\beta} = \sum_{i,j} r(\bar{y}_{i.j} - \bar{y}_{i..} - \bar{y}_{..j} + \bar{y}_{...})^2 \qquad \text{with } \mathsf{df}_{\alpha\beta} = (l_1 - 1)(l_2 - 1)$$

$$\mathsf{SSB} = \sum_{i,t} l_2 (\bar{y}_{it.} - \bar{y}_{...})^2 \qquad \text{with } \mathsf{df}_{B} = l_1 r - 1$$

With these sum of squares and degrees of freedom we may construct the F-statistics for the tests of $H_0: \ \boldsymbol{\beta} = 0$ or $H_0: \ \boldsymbol{\alpha\beta} = 0$ in the usual way.

ANOVA decomposition (cont.)

In the whole-plot stratum the block sum of squares SSB are further decomposed. The total sum of squares of the whole-plot stratum are the SSB of the split-plot stratum:

$$\mathsf{TSS}_{whole} = \mathsf{SSB} = \mathsf{SST}_\alpha + \mathsf{SSE}_{whole}$$

$$\mathsf{SST}_\alpha = \sum_{i=1}^{l_1} l_2 r(\bar{y}_{i..} - \bar{y}_{...})^2 \qquad \text{with } \mathsf{df}_\alpha = l_1 - 1$$

$$\mathsf{SSE}_{whole} = \sum_{i,t} l_2(\bar{y}_{it.} - \bar{y}_{i..})^2 \qquad \text{with } \mathsf{df}_E = l_1(r-1)$$

The F-statistic appropriate for testing $H_0: \ \alpha = 0$ is formed with the above sum of squares and degrees of freedom.

estimable contrasts

All linear combinations of the elements of $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\alpha\beta}$ are estimable.

$$\widehat{\mathbf{c}^T\boldsymbol{\alpha}} = \mathbf{c}^T\hat{\boldsymbol{\alpha}} = \frac{1}{l_1}\mathbf{c}^T\mathbf{F}^{\alpha T}\begin{pmatrix} \bar{y}_{1..} \\ \bar{y}_{2..} \\ \vdots \\ \bar{y}_{l_1..} \end{pmatrix}$$

The variance of this linear combination was given some slides above. With this we compute $(1-\alpha)$-confidence intervals:

$$\widehat{\mathbf{c}^T\boldsymbol{\alpha}} \pm \mathsf{t}_{1-\frac{\alpha}{2}}(l_1(r-1))\sqrt{\frac{\mathbf{c}^T\mathbf{c}}{n}\mathsf{MSE}_{whole}}$$

estimable contrasts (cont.)

The linear combinations of the split-plot parameters and their confidence intervals are computed completely analogously

$$\widehat{\mathbf{c}^T\boldsymbol{\beta}} = \frac{1}{l_2}\mathbf{c}^T\mathbf{F}^{\beta^T} \begin{pmatrix} \bar{y}_{..1} \\ \bar{y}_{..2} \\ \vdots \\ \bar{y}_{..l_2} \end{pmatrix} \qquad \widehat{\mathbf{c}^T\boldsymbol{\alpha\beta}} = \frac{1}{l_1 l_2}\mathbf{c}^T\left(\mathbf{F}^{\beta} \otimes \mathbf{F}^{\alpha}\right)^T \begin{pmatrix} \bar{y}_{1.1} \\ \bar{y}_{1.2} \\ \vdots \\ \bar{y}_{l_1.l_2} \end{pmatrix}$$

The variances of these linear combinations are again given some slides above.

$$\widehat{\mathbf{c}^T\boldsymbol{\beta}} \pm \mathsf{t}_{1-\frac{\alpha}{2}}(l_1(r-1)(l_2-1))\sqrt{\frac{\mathbf{c}^T\mathbf{c}}{n}\mathsf{MSE}_{split}}$$

$$\widehat{\mathbf{c}^T\boldsymbol{\alpha\beta}} \pm \mathsf{t}_{1-\frac{\alpha}{2}}(l_1(r-1)(l_2-1))\sqrt{\frac{\mathbf{c}^T\mathbf{c}}{n}\mathsf{MSE}_{split}}$$

10.3 SPD(B,B)

A two-factor split-plot experiment can also be organized so that both the among-blocks and within-blocks components of the study are executed as CBDs.

Instead of randomly allocating $l_1 r$ days to l_1 levels of the first factor, the experiment is run in r replicates, each replicate executed during a different week.

Within a given replicate/week, l_1 days are randomly allocated to the l_1 levels of the hard-to-change, among-blocks factor. As a result, the top stratum of the design is a CBD in which the l_1 treatments are applied once each to the units (days) of each replicate/block (week).

The bottom stratum of the design is a CBD as before.

The potential advantage of a SPD(B,B) is the improvement in power and precision that might be attained in comparing levels of the whole-plot factor.

10.3.1 A model

An overparameterized model for this design is given by

$$y_{tij} = \rho_t + \zeta_{ti} + \dot{\alpha}_i + \dot{\beta}_j + (\dot{\alpha\beta})_{ij} + \varepsilon_{tij} \qquad i = 1, \ldots, l_1; \quad j = 1, \ldots, l_2; \quad t = 1, \ldots, r$$

with

$$\begin{array}{lll} \zeta_{ti} \sim iid & \mathsf{E}(\zeta_{ti}) = \mu_\zeta & \mathsf{Var}(\zeta_{ti}) = \sigma^2_\zeta \\ \varepsilon_{tij} \sim iid & \mathsf{E}(\varepsilon_{tij}) = 0 & \mathsf{Var}(\varepsilon_{tij}) = \sigma^2 \end{array}$$

The influence of the new blocking structure (weeks) is represented by ρ_t. This model has $(r-1)$ additional degrees of freedom compared to a SPD(R,B).

A matrix formulation of the full rank model parameterization for the experiment may be written as

$$\mathbf{y} = \mathbf{W}\boldsymbol{\rho} + \mathbf{X}_1\boldsymbol{\zeta} + \mathbf{X}_\alpha\boldsymbol{\alpha} + \mathbf{X}_\beta\boldsymbol{\beta} + \mathbf{X}_{\alpha\beta}\boldsymbol{\alpha\beta} + \boldsymbol{\varepsilon}$$

where $\boldsymbol{\rho}$ is an r-element vector of fixed-effect block parameters and

$$\mathbf{W} = \mathbf{1}_{l_2} \otimes \mathbf{I}r \otimes \mathbf{1}_{l_1}$$

A model (cont.)

The intra-block estimators of $\mathbf{c}^T\boldsymbol{\beta}$ and $\mathbf{c}^T\boldsymbol{\alpha\beta}$ are not affected by the addition of whole-plot blocks to the model because the columns of $\mathbf{W}$ (representing groups of whole-plot units or split-plot blocks) can be formed as linear combinations of the columns of $\mathbf{X}_1$. So the intra-block model is the same as with SPD(R,B).

The inter-block model must be modified to reflect the new stratum of blocking

$$\mathbf{y}_2 = l_1(\mathbf{I}_r \otimes \mathbf{1}_{l_1})\boldsymbol{\rho} + l_2(\mathbf{1}_r \otimes \mathbf{F}^{\alpha})\boldsymbol{\alpha} + \boldsymbol{\varepsilon}^*$$

This is just a model for a CBD with r blocks, each containing l_1 units distributed among the l_1 levels of the factor associated with $\boldsymbol{\alpha}$.

Because CRDs and CBDs that each assign r units to each of l_1 treatments are Condition E-equivalent, the inter-block estimates are also not affected by the addition of whole-plot blocks.

10.3.2 Analysis

The estimates of treatment-related parameters have the same form whether orthogonal blocking is used at the whole-plot stratum or not.

But the additional structure must be taken into consideration in the ANOVA decomposition.

The sum of squares decomposition of the split-plot stratum is exactly as with SPD(R,B).

The whole-plot stratum has just an additional sum of squares corresponding with the new block parameters

$$\mathsf{TSS}_{whole} = \mathsf{SSB}_{split} = \mathsf{SST}_{\alpha} + \mathsf{SSB}_{whole} + \mathsf{SSE}_{whole}$$

$$\mathsf{SSB}_{whole} = \sum_{t=1}^{r} l_1 l_2 r (\bar{y}_{t..} - \bar{y}_{...})^2 \qquad \text{with } \mathsf{df}_B = r - 1$$

The residual sum of squares SSE_{whole} should be smaller compared to SPD(R,B), but also the the error degrees of freedom decrease from $\mathsf{df}_E = l_1(r-1)$ in the SPD(R,B) to $\mathsf{df}_E = (l_1 - 1)(r-1)$ in the SPD(B,B)

10.4 More than two experimental factors

More than one factor may be applied at either or both strata in experiments designed to investigate more than two factors.

Suppose two factors (e.g. temperature and binder fiber) are held constant within each random block, and only the levels of one factor (e.g. stirring rates) are varied within blocks.

In this case, the temperature and binder fiber main effects and the temperature-by-binder fiber interaction would be analyzed as whole-plot effects (i.e., compared to whole-plot residual sum of squares).

The factorial effects tested against split-plot residual variation would be the stirring rates main effect, and all interactions involving this factor.

In general: A factorial effect appears in the whole-plot section if *all* involved factors are applied at the whole-plot stratum (i.e., change levels only between random blocks), and appears in the split-plot section if *any* involved factor is applied at the split-plot stratum.

10.5 More than two strata of experimental units

Split-plot designs can often be identified easily because the experimental material is organized in a nested structure, and different factors or groups of factors are applied at different strata of that structure.

In some experiments, the available experimental material has a more extensive hierarchical structure involving more than two strata.

The „general rule“ for determining the organization of a two-stratum split-plot ANOVA (on the slide before) can be easily generalized to an arbitrary number of hierarchical strata of blocking.

E.g. in a three-stratum system of units, if factor A is applied at the top (largest) stratum, and factors B and C are applied at the intermediate stratum, then the associated three-factor interaction is tested in the intermediate section of the table.

Experiments performed in previous courses

This is an incomplete list of experiments (and topics) carried out in earlier installments of this and similar lectures.

- Percentage of popped kernels in microwave popcorn (PBIBD)
- Speed of enzyme inhibition (ODE)
- Injection moulding (RSD)
- Lifetime of constructions in a wind exposed environment (SPD)
- Distance of tennis shots (ODE, RSD)
- Duration of a pendulum (FFD)
- The amount of foam on beer (CBD)
- Growth of watercress (SPD)
- Many variants of Box's paper helicopter experiment

Don't replicate, be inventive! This is solely for your inspiration.