Advice for difficult homework exercises Experimental Design #5

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- **ad 13:** In the textbook (where this exercise is from) there is an apparent typo in exercise 7 of chapter 6. Of course it should read as follows:

  - Generate $n_i = 10$ data values from normal distributions with $\mu_i = 10 \cdot i$ and $\sigma_i^2 = \sqrt{10 \cdot i}$, $i = 1, \cdots, 5$

  power transformations are designed for data where $\mathsf{Var}(y) \approx (\mathsf{E}(y))^q$. Here you should simulate data for the case $q = \frac{1}{2}$. So from theory we know $p = \frac{2-q}{2} = \frac{3}{4}$ should be a good choice. But do as if you don't know and estimate p as requested in exercise 13 ...

- **ad 16:** I have problems to see a problem at all in this exercise ...

  I'm sure you all have noticed that the „experimental“ situation is a little bit strange:

  - from the experimental point of view we would expect the units to be different radon detectors and treatments e.g. different radon concentrations, temperatures etc.
  - from the design of experiment point of view the different detector types are the treatments.

  How to compute the block-sum-of-squares SSB, treatment-sum-of-squares SST, error-sum-of-squares SSE and total-sum-of-squares SStot you should have learned in a basic lecture about linear models:

  - you get the (corrected) SStot simply by multiplying the variance of the responses by $(n-1)$
  - for SSE you have to fit the full model with blocks and treatments

    $$y_{ij} = \alpha + \beta_i + \tau_j + \varepsilon_{ij} \qquad i = 1, \ldots, b;\ j = 1, \ldots, t$$

    SSE is just the residual sum of squares of this model.
  - for SST you have to fit the submodel with only blocks

    $$y_i = \alpha + \beta_i + \varepsilon_i \qquad i = 1, \ldots, b$$

    SST is just the difference between the residual sum of squares of this submodel and SSE and

    SSB is just the difference between SStot and the residual sum of squares of this submodel.

It should be obvious how to get the degrees of freedom associated with the above sum of squares.

You have to test the null hypothesis H_0 : *no treatment effects* (the mathematical formulation is left to you). The F-statistic for this test probably has something to do with the treatment mean squares MST and the MSE.

For the linear contrasts you have to compute the estimators $\boldsymbol{\tau}$ on the basis of a simple linear regression of the response vector on $\mathbf{X}_{2|1}$. You may find in the slides or the textbook the self-evident relation $\widehat{\mathbf{c}^T\boldsymbol{\tau}} = \mathbf{c}^T\hat{\boldsymbol{\tau}}$. The standard error of these estimates is $\sqrt{\mathsf{MSE} \cdot \mathbf{c}^T\mathcal{I}^-\mathbf{c}}$.

I hope it comes out well ... You should really understand this exercise, there will be other exercises based on this or similar models.