Advice for difficult homework exercises
Experimental Design #4

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- **ad 11:** Part (b) could already help you ...

  The answer to (b) is „certainly YES“, so $\mathbf{X}_2$ and $\mathbf{H}_1\mathbf{X}_2$ have to be the same for both designs. We will come back to that later.

  Let us first construct the design matrices as if we would have a regular unreplicated Latin square with $(t+1)$ treatments. The nuisance parameter design matrix $\mathbf{X}_1$ doesn't change if we apply treatment 1 instead of treatment $(t+1)$ to all units originally dedicated to treatment $(t+1)$ - so also $\mathbf{H}_1$ doesn't change.

  Only $\mathbf{X}_2$ changes a little bit. Originally $\mathbf{X}_2$ consists of $(t+1)$ different permutation matrices for each row-block. If we apply treatment 1 to all units dedicated to treatment $(t+1)$ we have to place the one in column 1 instead in column $(t+1)$ in all rows associated to these units.

  In a more algebraic way we could say: we add the $(t+1)$-th column to the first column of the original $\mathbf{X}_2$ and cancel column $(t+1)$ afterwards (if we do not apply treatment $(t+1)$ it makes no sense to record a parameter for this treatment). If you order the rows of $\mathbf{X}_2$ with respect to the columns you should see that you get $\mathbf{X}_2$ from the CRD mentioned in part (b).

  Lets go on with the computations for $\mathbf{X}_{2|1}$

  - think about what the sum of this $(t+1)$ transformed permutation matrices is.
  - then try to compute $\mathbf{X}_1^T\mathbf{X}_2$
  - try to find a matrix $\mathbf{Z}$ of simple structure such that $\mathbf{X}_1^T\mathbf{X}_2 = \mathbf{X}_1^T\mathbf{Z}$ (very similar to what you can see on the bottom of page 81 in the textbook).

  Now we can see that $\mathbf{H}_1\mathbf{X}_2 = \mathbf{H}_1\mathbf{Z}$. $\mathbf{H}_1$ projects on the linear space spanned by the columns of $\mathbf{X}_1$. So what happens if you project the columns of $\mathbf{Z}$ onto this space?

  Now you should have $\mathbf{X}_{2|1}$. Try to compute $\mathbf{X}_{2|1}$ for the design mentioned in part (b) and compare the results.

  Condition E!!!