

FORMAL MODELING

How to Write a Mathematical Article



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Some general remarks

- Every author has her/his own style and preferences how to write an article.
- The structure strongly depends on the topic/message of the article.

⇒ **There is no final word how one should write a mathematical article...**

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The outline

1. The general form using latex (with the LNCS-style)

Some (hopefully) useful remarks (based on my experience):

2. How to write the mathematical content (just math!)
3. Presenting the main content (with explanations)
4. General/personal hints/tips

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1. The general form using latex (with the LNCS-style)

```
\documentclass[english, runninghead, a4paper]{lncs}
... %define macros/load extra packages...
\begin{document}

\title{Find a catchy title\thanks{Supported by the Austrian FWF...}}

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\institute{Research Institute for Symbolic Computation (RISC)\ Johannes Kepler University Linz\
  Altenberger Str.\ 69, 4040 Linz, Austria\ \email{cschneid@risc.jku.at}}
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\begin{abstract}
  The abstract should briefly summarize the contents; 150–250 words.
  \keywords{First keyword \ and Second keyword \ and Another keyword.}
\end{abstract}

\section{Introduction}\label{Sec:Introduction}

\section{Further sections...}\label{Sec:NextSection}

\section{Conclusion}\label{Sec:Conclusion}

\subsubsection*{Acknowledgements} I would like to thank ...

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1. The general form using latex (with the LNCS-style)

```
\documentclass[english, runninghead, a4paper]{llnccs}
... %What ever you need...
\begin{document}
```

The document style

- Each journal/conference proceeding has its own style whose rules one should/must follow.
- Warning: proceedings often have rigorous page limits!
- However, the structure (up to some technical details) is always the same.
- In the *PS Formal Modelling* we require the Springer style LNCS.

<https://resource-cms.springernature.com/springer-cms/rest/v1/content/19238648/data/v4>

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```

The title

- **The first hurdle:** 90% of the potential readers will check only the title.
- A good title should
 - be simple and direct;
 - be attractive and informative;
 - contain the most important keywords;
(domain/objects, problem/theorem/conjecture, solution/method);
 - be not too long or short (or cryptic);
 - avoid nonstandard abbreviations and unnecessary acronyms.
- Often the funding agency (e.g. the Austrian FWF) is mentioned via a footnote.

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The author(s) and the institution(s) with email(s)

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```
\end{abstract}
```

The abstract (and keywords)

- **The second hurdle:** Among the 10% who passed the title, 90% will quit after reading the abstract.
- Just plain text: avoid formulas, citations, and references to the text below.
- Simple sentences, avoid technical words (for the non-expert).
- Why should the reader continue reading: explain what you advertised with the title. What is special?
- Summarize the main content/message of the article.

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1. The general form using latex (with the LNCS-style)

`\section{Introduction}`

The introduction (1–2 pages + ϵ)

- **The final hurdle:** Among the remaining readers only 10% will proceed...
- It should present the state-of-the-art citing relevant/related work.
- It should connect the content of this article with the known results
- It should specify the main result:
 - ideally, a precise statement/specification/problem description;
 - if too complicated, a simplified version in prose;
 - if possible, a concrete (short!) example to illustrate the result.
- The last paragraph should sketch the outline of the remaining sections:

The outline of the article is as follows. In Section~\ref{Sec:NextSection} the basic definitions and the detailed problem description is given.... The conclusion is given in Section~\ref{Sec:Conclusion}.

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1. The general form using latex (with the LNCS-style)

```
\section{Further sections...}
```

The main content

- The structure depends on the concrete context/the preferences of the author.
- Some hints will be given later...
- Of great help are environments (available in LNCS):

We follow the definitions given in~\cite{PS:19}.

```
\begin{definition}\label{Def:DR}
```

A difference ring $(\set A, \sigma)$ is a ring $\set A$ equipped with a ring automorphism $\sigma: \set A \to \set A$.

```
\end{definition}
```

```
\begin{lemma}\label{Lemma:FirstResult}
```

...

```
\end{lemma}
```

Using Lemma~\ref{FirstResult} we can now prove our first main result.

```
\begin{theorem}
```

Article writing is fun.

```
\end{theorem}
```

- and macros (see, e.g., <https://arxiv.org/>)

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1. The general form using latex (with the LNCS-style)

`\section{Conclusion}`

The conclusion

- It summarizes the main results of the article (extra notions are allowed that occur within the main text).
- However, readers might jump directly from the introduction to the conclusion. Thus avoid too many technical words.
- An outlook is given. E.g., the following questions can be addressed:
 - What can be done next?
 - What are possible (future) applications?
 - Are there conjectures/unanswered problems?
 - How can the presented ideas be generalized/extended?

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\subsubsection*{Acknowledgements} I would like to thank ...
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The acknowledgment

- This part is optional.
- If appropriate, one can thank
 - colleagues (advisors) for fruitful discussions;
 - referees for careful reading and valuable comments;
 - funding agencies (if the thanks command in the title is excluded by the editor).

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```

The bibliography

- Store all bib-items in `mybibliography.bib`. E.g.,

```
@incollection {PS:19,  
  AUTHOR = {Paule, Peter and Schneider, Carsten},  
  TITLE = {Towards a symbolic summation theory for unspecified sequences},  
  BOOKTITLE = {Elliptic integrals, elliptic functions and modular forms in quantum field theory},  
  SERIES = {Texts Monogr. Symbol. Comput.},  
  PAGES = {351–390},  
  PUBLISHER = {Springer, Cham},  
  YEAR = {2019}  
}
```

- Bib-items can be copied from many places:

- <https://mathscinet.ams.org/mathscinet/index.html>
- <https://dblp.uni-trier.de/>
- <https://zbmath.org/authors/>

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\appendix  
\section{Appendix}  
...  
\section{Appendix 2}  
...
```

The appendix

- This part is optional.
- Depending on the convention, it can be put before or after the bibliography.
- Move aspects to the appendix that would destroy the main flow of the article, but which are relevant for the interested reader. Typical examples are:
 - A detailed proof of a statement that is needed in the article (but which is irrelevant to understand the main result).
 - A detailed example/demo.
 - Further aspects that are related but that do not fit directly to the main topic.

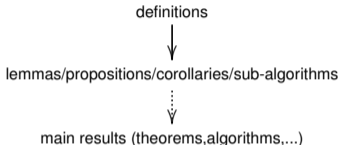
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2. **How to write the mathematical content (just math!)**
3. Presenting the main content (with explanations)
4. General/personal hints/tips

2. How to write the mathematical content (just math!)

[Note: here you see my personal opinions/experiences]

1. First produce correct mathematics in bottom-up style:



2. Afterwards check (and, if necessary, go back to 1):

- correctness;
- completeness:
 - Are there details missing in the proof?
→ Maybe an extra lemma is in place?
 - Can one easily generalize some results (without extra pain)?
 - Are all notions properly defined?
→ Insert extra definitions...
- Can one simplify the proofs?
 - Often the definitions are clumsy.
 - Being slightly less general might simplify the proofs dramatically (but sometimes the opposite is true...).

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3. Presenting the main content (with explanations)

There are two main approaches.

1. proof oriented: bottom-up construction

definitions \rightarrow lemma \rightarrow ... \rightarrow lemma/proposition \rightarrow main theorems/algorithms

2. reader friendly: top down construction

problem specification \rightarrow theorem \rightarrow lemma/proposition \rightarrow ... \rightarrow basic constructions/definitions

The reader should be motivated: why do we need all these technical proofs/constructions to obtain the main result?

Problem: (1) is rather technical, but (2) is often not feasible.

3. Presenting the main content (with explanations)

Often the following combined/hybrid version (or variants of it) is appropriate:

1. Introduction: specify the problem/state the main results. If this is too involved, present simplified versions in text form; use examples for illustration.

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1. Introduction: specify the problem/state the main results. If this is too involved, present simplified versions in text form; use examples for illustration.
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3. Finally, apply the bottom-up approach in the remaining section(s).
But in the beginning of each section describe in simple words, what one will accomplish and how this is related to the original problem.

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But in the beginning of each section describe in simple words, what one will accomplish and how this is related to the original problem.

Rather time consuming, but it pays off (concerning readability):

Construct one (or more) running example(s) that illustrate the full construction

- it should be compact but not trivial (the highlights/results of the article must be illustrated);
- it should start with the basic definitions and end, e.g., with the output of your algorithm. 26/29

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- **The devil is in the detail, or: typos prove the existence of the devil!**
 - Read the text many times (loud).
 - Put the text aside and read it some days later.
 - Check the text with different focuses:
 - just parse formulas for syntactical correctness (brackets etc);
 - read the text but ignore formulas;
 - compare related formulas and check inconsistencies;
 - check if all objects are defined when they arise the first time;
 - check if all existing results are properly cited;
 - check the citation list (spelling);
 - check if you cited all articles that are relevant;
 - ...

Some general remarks

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- The structure strongly depends on the topic/message of the article.

⇒ **There is no final word how one should write a mathematical article...**

Good luck for an excellent article!