Assignment 3 for Formal Modeling (Summer Semester 2024)

Symbolic Summation and the modeling of sequences
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Deadline: June 26, 2024

For this assignment carry out one of the following two tasks.

Task 1 Implement polynomial summation.
Let $\mathbb{Q}[x]$ be the polynomial ring in the variable x with coefficients form the rational function field $\mathbb{Q}$. Implement two functions $\texttt{PolynomialTelescoping}_i(f)$ where both produce for the input $f \in \mathbb{Q}[x]$ the output $g \in \mathbb{Q}[x]$ with

$$g(x+1) - g(x) = f(x). \tag{1}$$

1. For the function $i = 1$ use the basis transformation between the standard basis and the falling factorial basis.
2. For the function $i = 2$ use the degree bounding lemma and solve the underlying system of linear equations.

Finally, implement the corresponding functions $\texttt{PolynomialSummation}_i(f, a, b)$ which computes

$$\sum_{x=a}^{b} f(x)$$

by using the solution $g(x)$ of (1) computed by $\texttt{PolynomialTelescoping}_i(f)$.
Your implementation can be carried out with your preferred computer algebra system (like Mathematica, Maple or Sage). The following files should be submitted:

1. A file with your documented code
2. together with file that contains concrete examples that can be carried out in your chosen CA system (e.g., a Mathematica notenook if you choose Mathematica); in particular, the efficiency should be compared between both methods.
3. A pdf-file where the computations (examples) from step 2 are documented explicitly.

Task 2: Implement the basic functionality of the term algebra SumProd($\mathbb{Q}(x)$).
Let $\mathbb{G} := \mathbb{Q}(x)$ be the rational function field in the variable x defined over the rational numbers $\mathbb{Q}$.

1. Implement the term algebra SumProd($\mathbb{G}$) together with the evaluation function ev : SumProd $\times \mathbb{N} \to \mathbb{Q}$ from Section 2. More precisely, given $A \in$ SumProd($\mathbb{G}$) with expressions as defined in Section 2 and given $n \in \mathbb{N}$, your function ev(A, n) should evaluate the expression accordingly.

2. Furthermore implement the function

$$\texttt{ReducedRepresentation} : \text{SumProd}(\mathbb{G}) \to \text{SumProd}(\mathbb{G})$$

   which transforms an expression from SumProd($\mathbb{G}$) to the reduced representation; see Definition 2.8. Note: a transformation to the sum-product reduced representation is not necessary.

Your implementation can be carried out with your preferred computer algebra system (like Mathematica, Maple or Sage). Hint: for the implementation of `ReducedRepresentation` you are allowed to use the standard field/ring operations (together with functions working on variables) of your available computer algebra system.

The following files should be submitted:

- A file with your documented code
- together with file that contains concrete examples that can be carried out in your chose CA system (e.g., a Mathematica notenook if you chose Mathematica); in particular, the correctness of the evaluations should be demonstrated.
- A pdf-file where the computations (examples) from step 2 are documented explicitly.